Generalized Rigidity and Stability Analysis on Formation Control Systems

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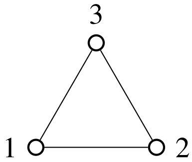
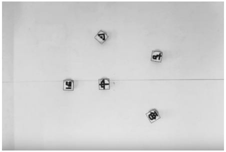
Multi-agent formation

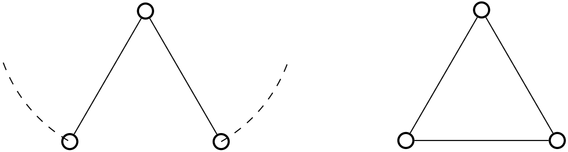
- A geometrical shape formed by multiple agents in a space.
- Represented by a graph (vertices=agents).
- Examples:
  - A group of ground vehicles,
  - A group of flying multi-copters.





Condition for unique formation shape under (distance) rigidity

- (Distance) rigidity is a condition in order that the formation of interest is uniquely defined under given distance constraints.
- (Distance) rigidity has been widely studied in the literature.[1]



(a) Flexible formation. (b) Rigid formation.

[1] L. Asimow and B. Roth. "The rigidity of graphs". In: *Transactions of the American Mathematical Society* 245.11 (1978), pp. 279–289; B. Hendrickson. "Conditions for unique graph realizations". In: *SIAM Journal on Computing* 21.1 (1992), pp. 65–84.

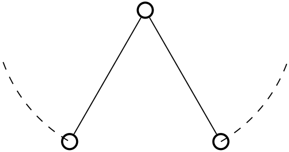
Condition for unique formation shape under bearing rigidity

- Bearing rigidity is a condition in order that the formation of interest is uniquely defined under given bearing constraints.
- Bearing rigidity has been studied in the literature.[2]

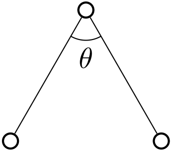
[2] S. Zhao and D. Zelazo. “Bearing rigidity and almost global bearing-only formation stabilization”. In: *IEEE Transactions on Automatic Control* 61.5 (2016), pp. 1255–1268.

Condition for unique formation shape under weak rigidity

- Rigidity with distance- and angle-constraint is named *weak rigidity*.
- Weak rigidity is a condition in order that the formation of interest is uniquely defined under given distance and angle constraints.
- Weak rigidity has been recently studied in the literature.[3]



(a) Flexible formation.

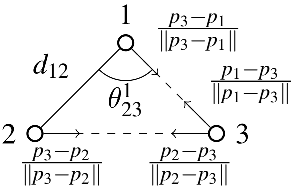


(b) Weak rigid formation.

[3] Myoung-Chul Park, Hong-Kyong Kim, and Hyo-Sung Ahn. "Rigidity of Distance-based Formations with Additional Subtended-angle Constraints". In: *Proc. of the 17th International Conference on Control, Automation and Systems (ICCAS)*.

Contribution: *Generalized rigidity*

- Rigidity with distance-, angle- and bearing-constraint is named *generalized rigidity*.
- Generalized rigidity is a condition in order that the formation of interest is uniquely defined under given distance, angle and bearing constraints.



(a) Triangular formation characterized by 1-distance, 2-bearing and 1-angle constraints.

Motivation

- To design a formation control system in networks with heterogeneous measurements.
- To avoid phenomena on flip ambiguity.

Example of flip ambiguity

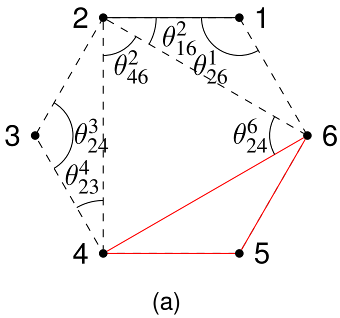


Figure. Hexagonal formation characterized by 4-distance and 6-angle constraints.

Notations and terminologies

- For a given graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V}$ is the vertex set, $\mathcal{E}$ is the edge set, and $\mathcal{A}$ is the angle set.
- d_{ij}: distance constraint assigned to $(i, j) \in \mathcal{E}_D$.
- g_{ij}: bearing constraint assigned to $(i, j) \in \mathcal{E}_B$.
- $\mathcal{E}_D \cap \mathcal{E}_B = \emptyset$ and $\mathcal{E} = \mathcal{E}_D \cup \mathcal{E}_B$.
- θ_{ij}^k: angle constraint assigned to $(k, i, j) \in \mathcal{A}$.
- $\mathbf{p}_i \in \mathbb{R}^d$: position vector of agent i.
- $\mathbf{p} \triangleq (\mathbf{p}_1, \ldots, \mathbf{p}_n) \in \mathbb{R}^{dn}$ is called a *configuration* in $\mathbb{R}^d$.
- $(\mathcal{G}, \mathbf{p})$ is called a *framework (formation)*.
- $\mathbf{z}_{ij} \triangleq \mathbf{p}_i - \mathbf{p}_j$: relative position.

Infinitesimal generalized rigidity

- The *generalized rigidity function* $F_G : \mathbb{R}^{dn} \rightarrow \mathbb{R}^{(m_D+w+dm_B)}$ is defined as

$$F_G(\mathbf{p}) \triangleq [\|\mathbf{z}_1\|^2, ..., \|\mathbf{z}_{m_D}\|^2, A_1, ..., A_w, g_1^\top, \cdots, g_{m_B}^\top]^\top$$

  where $m_D = |\mathcal{E}_D|, m_B = |\mathcal{E}_B|, w = |\mathcal{A}|$ and $A_h = \cos\theta_h$. The rigidity function describes constraints of distances, angles and bearings in given framework.

- The *generalized rigidity matrix* is defined as the Jacobian of the rigidity function as follows

$$R_G(\mathbf{p}) \triangleq \frac{\partial F_G(\mathbf{p})}{\partial \mathbf{p}} \in \mathbb{R}^{(m_D+w+dm_B)\times dn}.$$

Infinitesimal generalized rigidity

- If $\dot{F}_G = R_G(\mathbf{p})\delta\mathbf{p} = 0$, then $\delta\mathbf{p}$ is called an infinitesimal motion of $(\mathcal{G}, \mathbf{p})$.

Definition (Trivial infinitesimal generalized motion)

An infinitesimal motion of a framework $(\mathcal{G}, p)$ is called *trivial* if it corresponds to a rigid-body translation of the entire framework[4].

Definition (Infinitesimal generalized rigidity)

A framework $(\mathcal{G}, p)$ is *infinitesimally generalized rigid* in $\mathbb{R}^d$ if all of infinitesimal generalized motions are trivial.

[4] I do not explain the case of $\mathcal{E}_{\mathcal{D}} = \emptyset$ to avoid confusion.

Infinitesimal generalized rigidity

Theorem

A framework $(\mathcal{G}, p)$ with $n \geq 3$ and $\mathcal{E}_{\mathcal{D}} \neq \emptyset$ is infinitesimally generalized rigid in $\mathbb{R}^d$ if and only if the generalized rigidity matrix $R_G(p)$ is of rank $dn - d$[5].

- Rank condition of $R_G(p) \Rightarrow$ infinitesimally generalized rigid

[5] Infinitesimal motions correspond to rigid-body translations of entire framework.

Its application to formation control problem

- We assume that each agent is governed by a single integrator.
- Gradient flow law is employed as follows

$$\dot{\mathbf{p}} = u \triangleq -R_G(\mathbf{p})^\top \mathbf{e}(\mathbf{p}). \tag{1}$$

Theorem

Assume that a desired formation p^* is infinitesimally generalized rigid, and initial formation $p(0)$ is sufficiently close to the desired formation. Then, under the system (1), $p = p^*$ is asymptotically stable.

Assumption

Agents to measure bearings have a global reference frame.

Desired formation

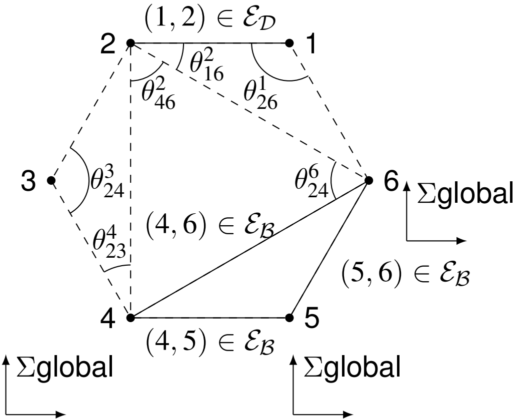
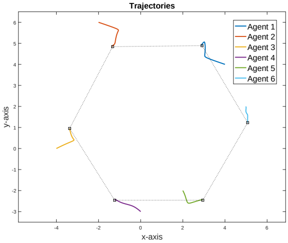
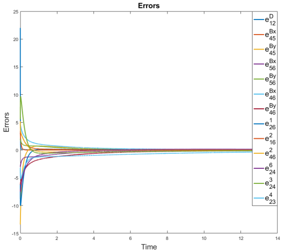


Figure. Graph for desired formation with 1-distance, 3-bearing and 6-angle constraints in $\mathbb{R}^2$.

Simulation



(a) Trajectories of 6 agents.



(b) Errors.

Figure. Numerical simulation on a 6-agent formation.

Summary

- Generalized rigidity using distance, angle and bearing constraints.
- Formation control with the generalized rigidity.
  - we can design a formation control system in networks with heterogeneous measurements.
  - we can avoid phenomena on flip ambiguity.
- Future work
  - To design a control system without the assumption that agents to measure bearings have a global reference frame.

References I

[1] L. Asimow and B. Roth. "The rigidity of graphs". In: *Transactions of the American Mathematical Society* 245.11 (1978), pp. 279–289.

[2] B. Hendrickson. "Conditions for unique graph realizations". In: *SIAM Journal on Computing* 21.1 (1992), pp. 65–84.

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Thanks for your attention.