Leader-Follower Bearing-based Formation System with Exogenous Disturbance

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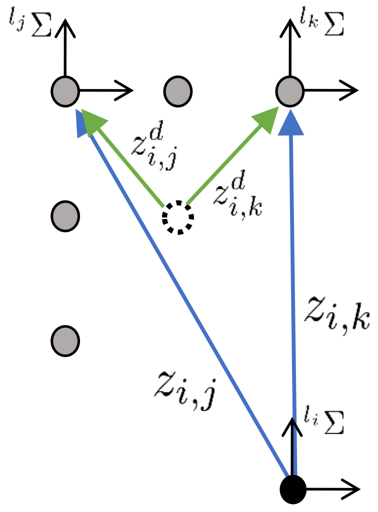
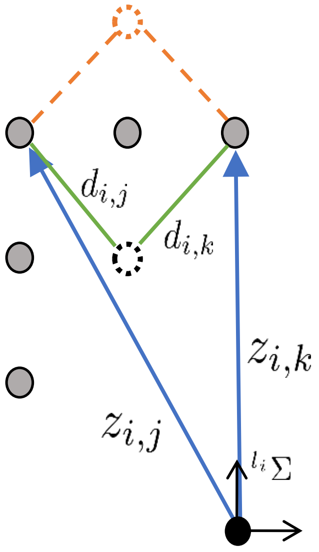


$$z_{i,j} \rightarrow z_{i,j}^d$$

$$z_{i,k} \rightarrow z_{i,k}^d$$

- Controlled variable: relative position.
- Sensed variable: relative position.
- Coordinate system: local coordinate frame with aligned orientation.
- Sensing capability: intermediate.
- Control difficulty: intermediate.

Distance-based

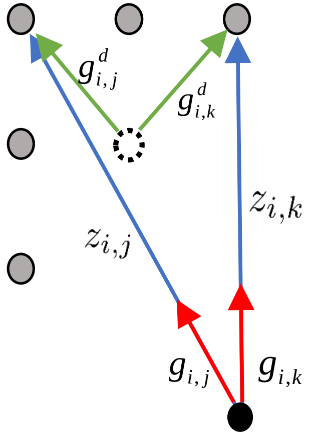


$$\|z_{i,j}\| \rightarrow d_{i,j}$$

$$\|z_{i,k}\| \rightarrow d_{i,k}$$

- Controlled variable: inter-agent distance.
- Sensed variable: relative position.
- Coordinate system: local coordinate frame.
- Sensing capability: low.
- Control difficulty: high.

Bearing-based



$$g_{i,j} = \frac{z_{i,j}}{\|z_{i,j}\|}$$

$$g_{i,j} \rightarrow g_{i,j}^d$$

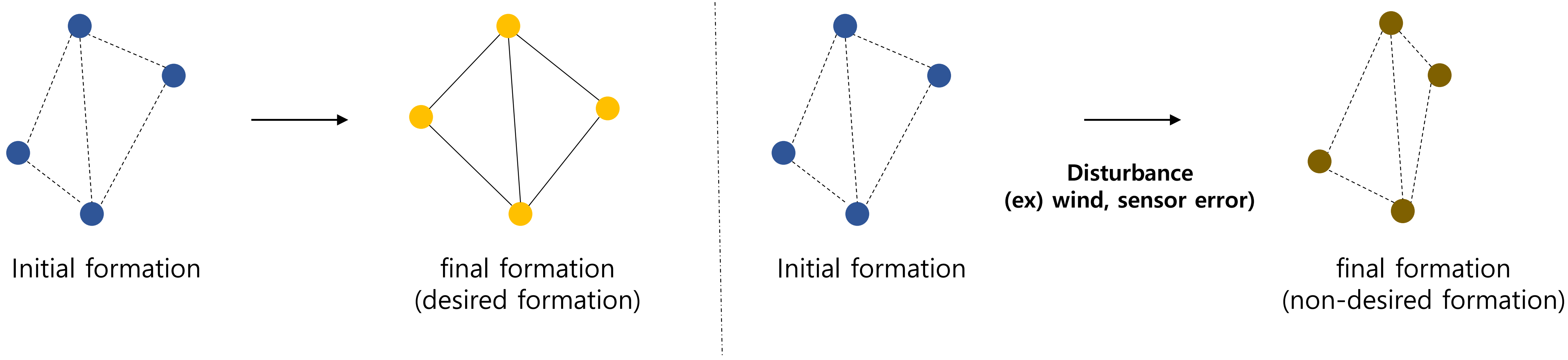
$$g_{i,k} \rightarrow g_{i,k}^d$$

- Controlled variable: relative bearing.
- Sensed variable: relative bearing.
- Coordinate system: global coordinate frame or local coordinate frame.
- Sensing capability: low.
- Control difficulty: high.

[1] Oh, Kwang-Kyo, Myoung-Chul Park, and Hyo-Sung Ahn. "A survey of multi-agent formation control." *Automatica* 53 (2015): 424-440.
[2] Zhao, Shiyu, and Daniel Zelazo. "Bearing rigidity and almost global bearing-only formation stabilization." *IEEE Transactions on Automatic Control* 61.5 (2015): 1255-1268.

Motivation

[Control goal]



- **Bearing formation error** – difference between desired formation and final formation.
- **Research goal** – analysis on *bearing formation error* in the presence of the exogenous disturbance.

Notation

- Interaction graph *([3])*
  - Let $G = (V, E)$ be an undirected graph of leader-follower network.
  - Vertex : agent, $V = \{1, 2, \ldots, n\}$, $|V| = n. (V = V_l \cup V_f, |V_l| = n_l, |V_f| = n_f, n = n_l + n_f)$.
  - Edge : interaction between agent, $E = \{(i, j) \mid i,\ j \in\ V\}, |E| = m$.
  - Neighborhood of agent : $N_i = \{\, j \in V \mid (i,\ j)\ \in\ E\,\}$.
  - An arbitrary orientation can be given for each edge -> *Incidence matrix.* $H \in \mathbb{R}^{m \times n}$.

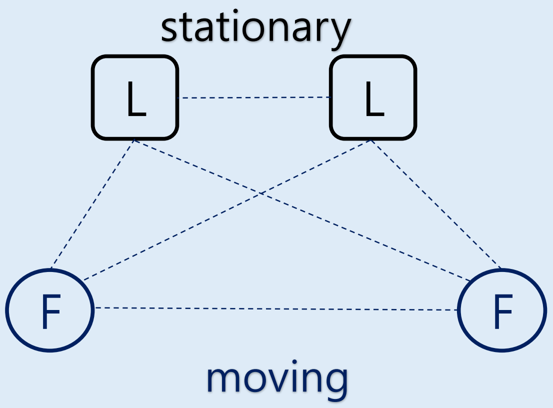


Figure: An example of leader-follower network.

- Framework
  - The formation is defined by a framework (G, p).
  - $G = (V, E)$, $p = [\, p_1^T,\ p_2^T,\ \ldots\ ,\ p_n^T\,]^T \in \mathbb{R}^{nd}$ is the position vector that maps each vertex to space.
  - $z_k = z_{k_{ij}} = p_j - p_i$ is the relative position vector and $g_k = z_k / \|z_k\|$ is the **bearing** vector of k^{th} edge between agents i and j.
  - For leader-follower formation, $p = [p_l^T\ ,\ p_f^T\,]^T \in \mathbb{R}^{nd}$, $p_l(t) = p_l^*$ (desired position of leader agent).

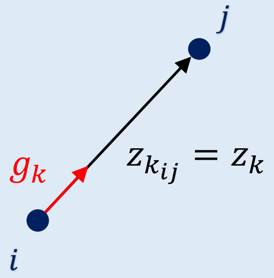


Figure: Relative position and bearing vector.

[3] Godsil, Chris, and Gordon F. Royle. *Algebraic graph theory.* Vol. 207. Springer Science & Business Media, 2013.

Bearing localizability

Definition 1 ([4]). The target formation is **bearing localizable** if the desired position p^* is uniquely determined by the desired bearing vector $\{g_{ij}^*\}_{(i,j)\in E}$ and the stationary position of the leader agent $\{p_i^*\}_{i\in V_l}$.

- From [4],
  - At least two leader agents exist to ensure the **bearing localizability.**
  - The *bearing Laplacian matrix B* for the target formation is

$$[B]_{ij} = \begin{cases} 0_{d\times d}, \ i \neq j, \ (i,j) \notin E \\ -P_{g_{ij}^*}, \ i \neq j, \ (i,j) \in E, \ \text{where } P_{g_{ij}^*} = I_d - g_{ij}^* g_{ij}^{*T} \\ \sum_{j\in N_i} P_{g_{ij}^*}, \ i = j, \ i \in V \end{cases} \rightarrow B = \begin{bmatrix} B_{ll} & B_{lf} \\ B_{fl} & B_{ff} \end{bmatrix} = \bar{H}^T diag(P_{g_k^*})\bar{H} \in \mathbb{R}^{dn\times dn}, \text{ where } \bar{H} = H \otimes I_d.$$

Lemma 1 ([5]). For the leader-follower bearing-based formation system, the target formation (G, p^*) is **bearing localizable** if and only if B_{ff} is positive definite.

[4] Zhao, Shiyu, Zhenhong Li, and Zhengtao Ding. "A Revisit to Gradient-Descent Bearing-Only Formation Control." *2018 IEEE 14th International Conference on Control and Automation (ICCA)*. IEEE, 2018.
[5] Zhao, Shiyu, and Daniel Zelazo. "Localizability and distributed protocols for bearing-based network localization in arbitrary dimensions." *Automatica* 69 (2016): 334-341.

Problem statement

Assumption 1. The target formation (G, p^*) is **bearing localizable** and at least two leader agents exist, i.e., B_{ff} is positive definite.

- Based on the *gradient-descent bearing-only control law* [4] , consider the leader-follower bearing-based formation system with the exogenous disturbance

$$\text{Leader dynamics - } \quad \dot{p}_i = 0, \; i \in V_l$$

$$\text{Follower dynamics - } \quad \dot{p}_i = u_i + f_i = \sum_{j \in N_i} (g_{ij} - g_{ij}^*) + f_i, \; i \in V_f \qquad (1)$$

where f_i is the exogenous disturbance vector of agent i.

Assumption 2. The exogenous disturbance vector is upper-bounded, i.e., $\|f_i\| \le v_i$, where v_i is a positive constant.

- From (1), the overall dynamics for the formation system is

$$\dot{p} = u + f = -\begin{bmatrix} 0 & 0 \\ 0 & I_{dn_f} \end{bmatrix} \bar{H}^T (g - g^*) + f, \qquad (2)$$

where g^* is the overall desired bearing vector and f is the overall exogenous disturbance vector.

[4] Zhao, Shiyu, Zhenhong Li, and Zhengtao Ding. "A Revisit to Gradient-Descent Bearing-Only Formation Control." *2018 IEEE 14th International Conference on Control and Automation (ICCA)*. IEEE, 2018.

Main result

- Define the **formation error** for the formation system as

$$e := p - p^*, \qquad (3)$$

where p^* is the desired position vector of the target formation (G, p^*).

- From (3), the **formation error dynamics** for the formation system is

$$\dot{e} = \dot{p}, \qquad (4)$$

which is equal to the position dynamics (2).

Lemma 1. Consider the leader-follower bearing-based formation system (2). The inter-agent distance between neigh boring agent is upper-bounded as

$$\|z_k(t)\| = \|p_i(t) - p_j(t)\| \leq 2ns(t), \ \forall t \geq 0 \qquad (5)$$

where i, j vertices connected by k^{th} oriented edge and $s(t)$ is the time-varying *scale*.

Assumption 3. The *scale* of the formation system is upper-bounded from the initial scale for all time with the exoge nous disturbance, i.e., $s(t) \leq s(0) = s_0, \forall t > 0$.

Main result

- Via the Lypuanov stability analysis, following theorem shows the **global exponential ultimate boundedness** of the *formation error* e in the presence of the exogenous disturbance.

Theorem 1. Consider the leader-follower bearing-based formation system (2) in the presence of the exogenous distur bance. The *formation error* e is **globally exponentially ultimately bounded** for the bounded exogenous disturbance. Furthermore, the *formation error* e globally exponentially converges to the bounded set S

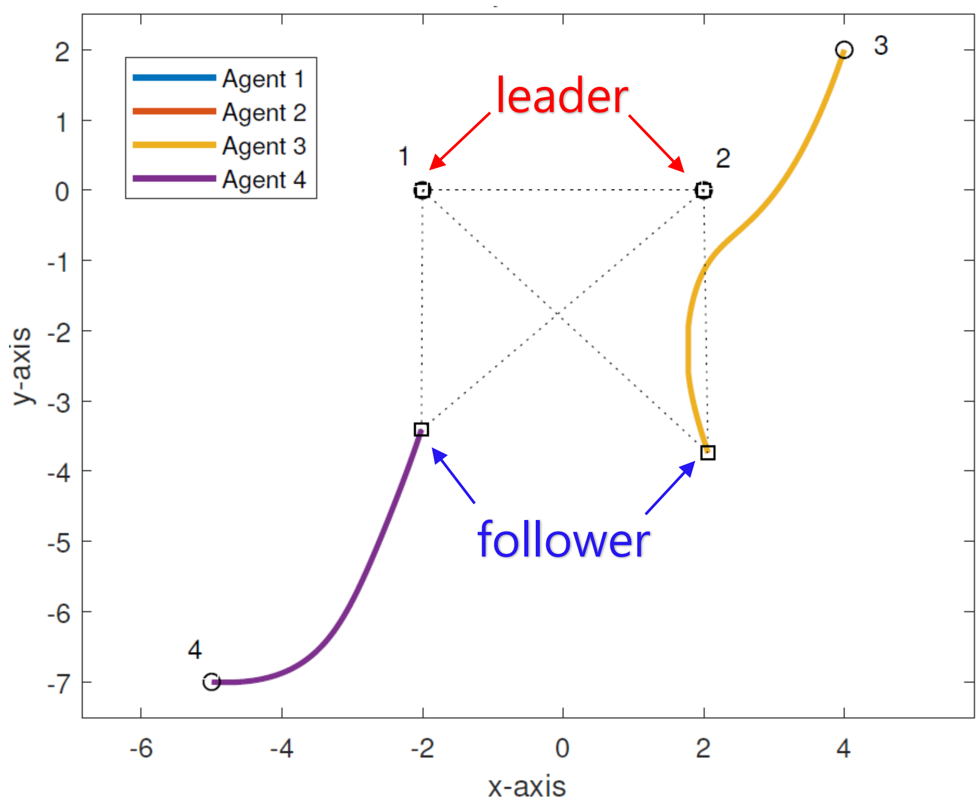
$$S = \left\{ e : \|e\|^2 \leq \frac{\gamma^2 \sum_{i=n_l+1}^{n} v_i^2}{\lambda_{\min}(B_{ff}) / 4ns_0 - \gamma^{-2} / 4 - \delta / 2} \right\}, \qquad (6)$$

where $\sum_{i=n_l+1}^{n} v_i^2$ is the upper-boundedness of disturbance and $\lambda_{\min}(B_{ff})$ is the minimum eigenvalue of B_{ff}.

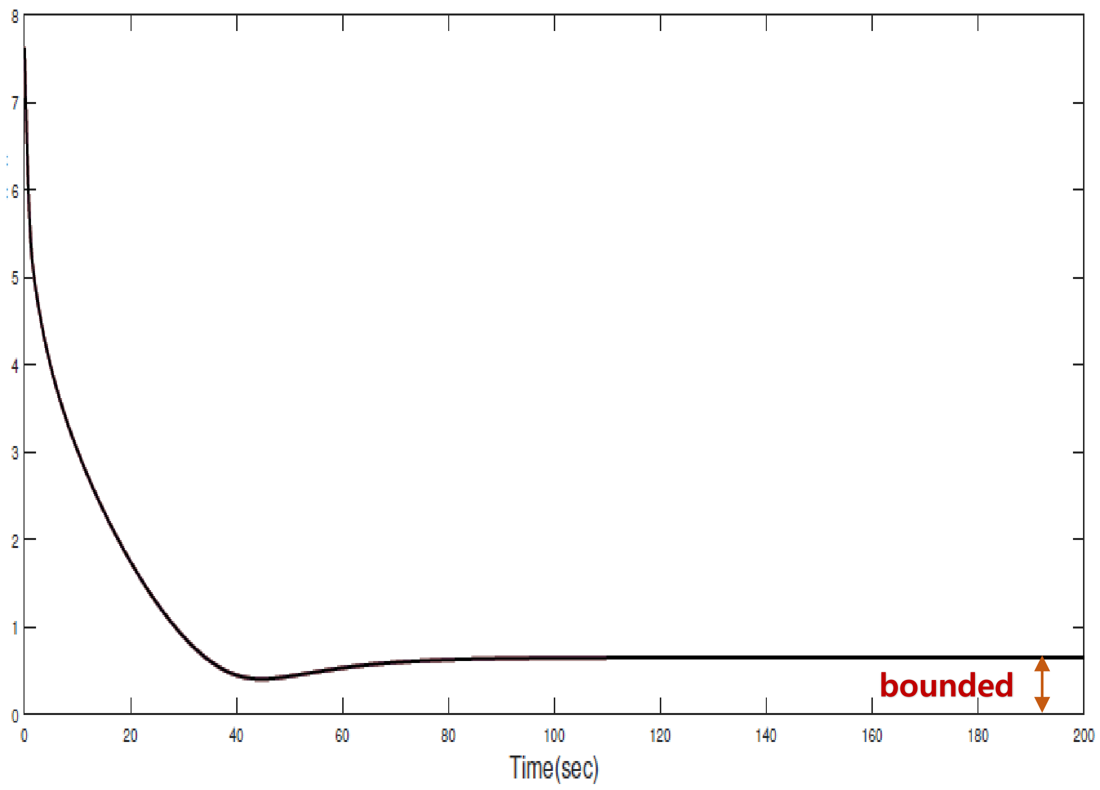
Remark 1. From (6), the upper-boundedness of the *formation error* e of the formation system is relevant to the exog enous disturbance, network topology of the formation, initial scale and system parameters.

Simulation

4-agent leader-follower formation system in two-dimensional space



Trajectory of the formation



Bounded formation error $||e||$

Conclusion

- We analyzed the leader-follower bearing-based formation system in the presence of the exogenous disturbance.
- Under the gradient-descent bearing-only formation control law, the **global exponential ultimate boundedness** of the *formation error* was shown.
- Via the Lyapunov stability analysis, the upper-boundedness of *formation error* was explicitly given.
- Simulation result validated the theoretical result.
- Interesting future work will be the stability analysis for leader-less bearing-based formation system in the presence of the exogenous disturbance.

Reference

- Oh, Kwang-Kyo, Myoung-Chul Park, and Hyo-Sung Ahn. "A survey of multi-agent formation control." *Automatica* 53 (2015): 424-440.
- Zhao, Shiyu, and Daniel Zelazo. "Bearing rigidity and almost global bearing-only formation stabilization." *IEEE Transactions on Automatic Control* 61.5 (2015): 1255-1268.
- Godsil, Chris, and Gordon F. Royle. *Algebraic graph theory*. Vol. 207. Springer Science & Business Media, 2013.
- Zhao, Shiyu, Zhenhong Li, and Zhengtao Ding. "A Revisit to Gradient-Descent Bearing-Only Formation Control." *2018 IEEE 14th International Conference on Control and Automation (ICCA)*. IEEE, 2018.
- Zhao, Shiyu, and Daniel Zelazo. "Localizability and distributed protocols for bearing-based network localization in arbitrary dimensions." *Automatica* 69 (2016): 334-341.

Thank you

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