Approach to the Problem of Localization with data loss

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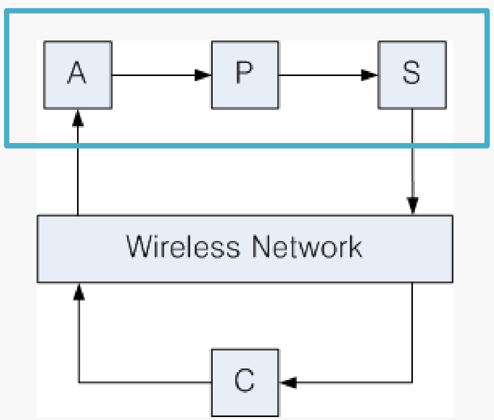
Problem of Ubiquitous Localization Network

- Imperfections in single control loop include
  - Delay and jitter
  - Bandwidth limitation
  - Data loss and bit errors
  - Outages and disconnection



Sensor

Actuator

Plant

Localization with Data Loss

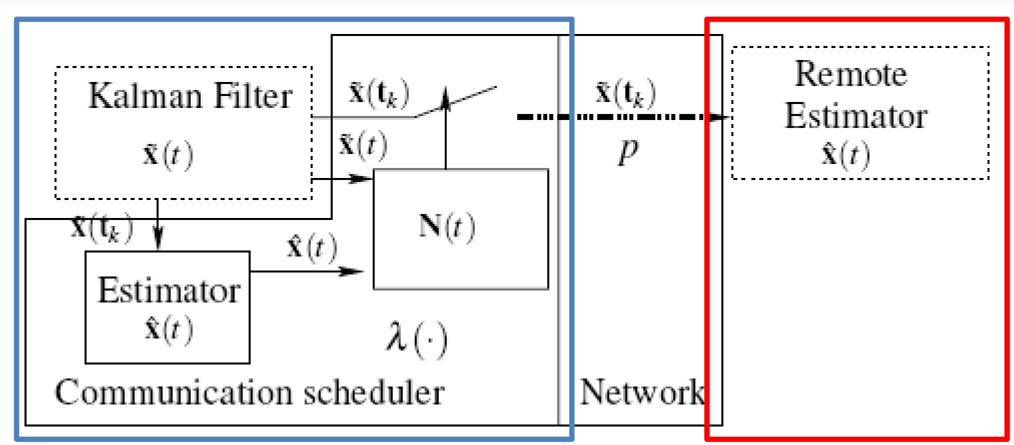
- Keyword
  - Lossy Network
  - Sample Rate Varying
  - Data Dropout
  - Data Loss
  - Intermittent Observations
  - Dropping Packets

- Implication Definition
  - Estimation with considering mobile target, sample rate varying, and distributed system.

Estimation under uncontrolled and controlled communication in NCS

- A communication pattern : a communication scheduler on the smart sensor and a lossy network

Smart Sensor



$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{w}$$
$$\mathbf{y} = C\mathbf{x} + \mathbf{v}$$

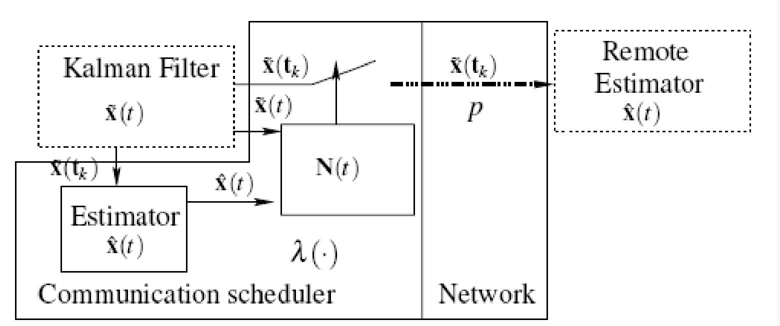
- Authors model data sending times as jumps of an integer random process (Poisson process).
- The intermittent time can be determined by controlled random processes (Poisson process) so as to use system dynamics information.

Estimation under uncontrolled and controlled communication in NCS

- An integer-valued random measure N(t) is constructed by Possion process mechanism.

$$\Pr\left[\mathbf{N}(t+dt) - \lim_{s\to t^-}\mathbf{N}(s) = 1\right] = \Lambda\left(\mathscr{I}(t^-)\right)dt$$

- $\Lambda\left(\mathscr{I}(t^-)\right)dt$ is probability of sending data.
- P is probability of dropping packets.



Estimation under uncontrolled and controlled communication in NCS

The communication pattern models the communication scheduling and the network uncertainties, as in Fig. 4. Specifically, we consider two patterns:

1) The communication scheduler is driven by a Poisson process with a constant Poisson rate λ, and the packets get lost with probability p, $0 \leq p < 1$. Since packet loss is independent of the Poisson process, data arrives at the remote estimator according to another Poisson process with rate $(1-p)\lambda$.
2) The communication scheduler is driven by an integer-valued process with jump intensity $\lambda(\tilde{\mathbf{e}}^-)$, and the packets get lost with probability p. In this case, the effective intensity becomes $(1-p)\lambda(\tilde{\mathbf{e}}^-)$.

For analysis purposes, the data loss probability p does not add complexity. From now on, λ or $\lambda(\tilde{\mathbf{e}}^-)$ refers to the intensity that data are *received* on the remote estimator.

Estimation under uncontrolled and controlled communication in NCS

- γ is constant Poisson rate

Consider a communication scheduler that is driven by a Poisson process with a rate $\gamma > 0$. Define

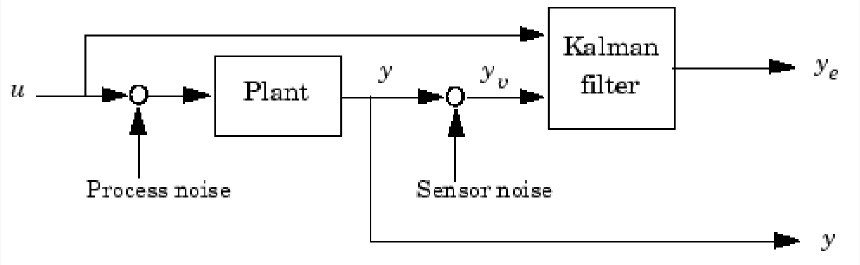
$$\gamma_{2m} := 2m \max\{\Re[\mathrm{Eig}(A)]\}.$$

Theorem 1. *Let the estimation error* $\hat{\mathbf{e}}$ *be defined as in* (1), (3)-(5), *in which the time sequence* $\mathbf{t}_k$, $k \geq 0$, *is generated by a Poisson process with a nonnegative rate* γ. *For any* $m \geq 1$, *if* $\gamma > \gamma_{2m}$, $\mathrm{E}[(\hat{\mathbf{e}}(t)'\hat{\mathbf{e}}(t))^m]$ *is bounded,* $\forall\, t \geq 0$, *and if* $\gamma < \gamma_{2m}$, $\lim_{t\to\infty} \mathrm{E}[(\hat{\mathbf{e}}(t)'\hat{\mathbf{e}}(t))^m] \to \infty$.

- γ_{2m} is a tight bound.
- In the first pattern, Estimation error covariance is bounded if P is larger than tight bound.

Time-Varying Kalman Filter

- In the related papers, raw measurements are sent to the remote TVKF via a lossy network.
- Plant state and measurement equations



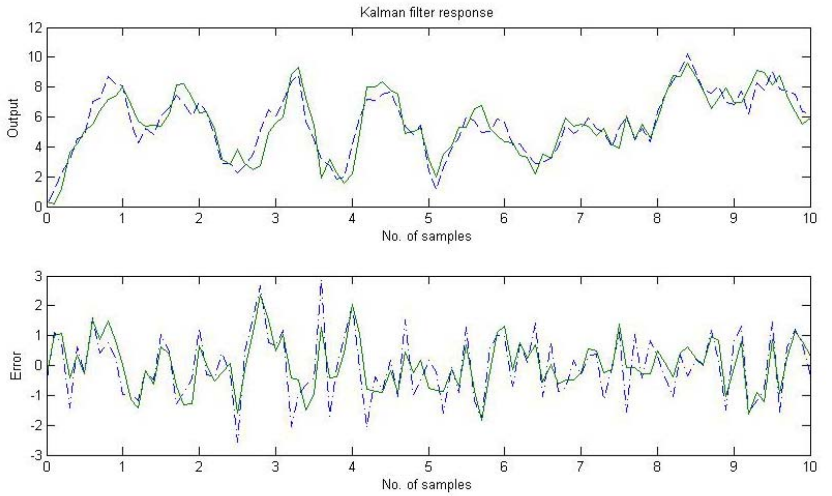
$$x[n+1] = Ax[n] + Bu[n] + Bw[n]$$
$$y_v[n] = Cx[n] + v[n]$$

- Time update
  - *State estimation uncertainty is considered by every time update.*

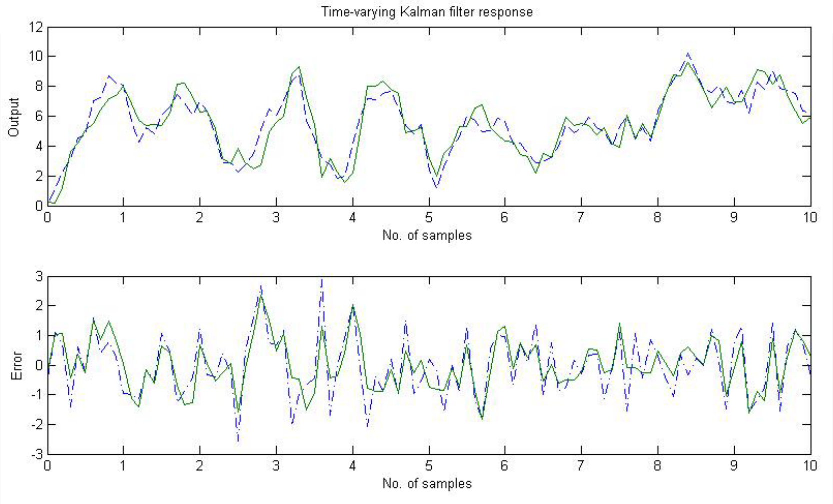
$$\hat{x}[n+1|n] = A\hat{x}[n|n] + Bu[n]$$
$$P[n+1|n] = AP[n|n]A^T + BQ[n]B^T$$

Time-Varying Kalman Filter

Steady State Design



Time-varying Design



Error Covariance

0.7232

0.7234

The value of TVKF is close to the value obtained for the steady state design.