Decentralized Iterative Learning Control for Heterogeneous System with Arbitrary Interconnections

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December 6, 2012

Summary

- Subunits share their interconnection signals with directly interconnected neighbors.
- Iterative learning controllers improve their control signal in a decentralized manner.
- Then the controllers make the outputs of the subunits to track the desired outputs on a fixed time in an iteratioin domain.

Background

- A significant amount of research efforts has been focused on control of spatially interconnected systems, due to its emerging applications in diverse areas.
- Fast steering mirrors and deformable mirrors have recentely been dealt as key applications of spatially interconnected systems.
- Control strategies such as H_∞ control and iterative learning control(ILC) which can be synthesized in the interconnected systems have advantages in terms of practicality, cost-effectiveness, high-performance.

Iterative Learning Control(ILC)

- Iterative learning control is a method to improve the performance of a system that operates repetitively on a finite time interval. For example, suppose that there is a ILC K which makes a given plant assymptotically stable. Then it can be expressed by

$$\lim_{i \to \infty} e_i(t) \to 0 \ \ \forall t = [0, T]. \tag{1}$$

- Such improvement can be achieved by taking error signals from previous iterations as its feedback signal.

$$u_{i+1}(t) = u_i(t) + Qe_i(t) \tag{2}$$

- A huge amount of researches has been concentrated on the ILC of the spatially interconnected systems with various applications.

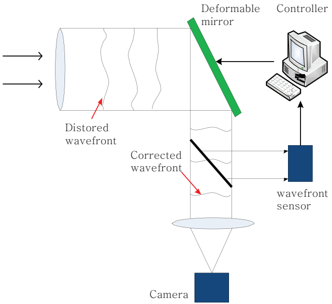

Figure: Adaptive optics system[1]

- Adaptive optics system is a control system to compensate wavefront errors of images.
- Such compensation can be acheived by using lots of actuators to deform the mirror.

[1] Hyo-Sung Ahn, Taekyung Lee, and Young-Soo Kim, "Iterative learning control for spatially inteconnected systems," 2011.

The selection of the deformable mirror type

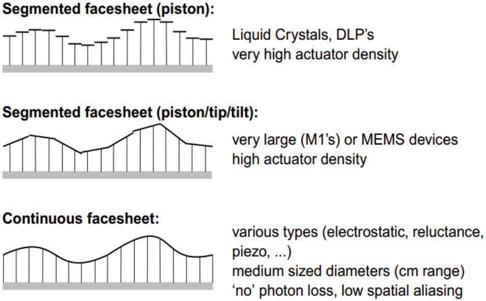

Figure: Three types of deformable mirrors

In this research, a deformable mirror is considered as the continuous facesheet type.

Using 2D plate theory, the deformable mirror can be expressed as

$$\sigma d \frac{\partial w^2}{\partial^2 t} + D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial^2 x \partial^2 y} + \frac{\partial^4 w}{\partial^4 y}\right) = u(t,x,y) \qquad (3)$$

where w denotes the deformation of the plate at (t,x,y), σ denotes the mass density, d denotes the thickness of the plate, $D = \left(\frac{Ed^3}{12(1-v^2)}\right)$ denotes the bending/flexural rigidity(E is the elastic modulus, and v denotes the poisson's ratio), and u denotes the distributed preessure generated from actuators.

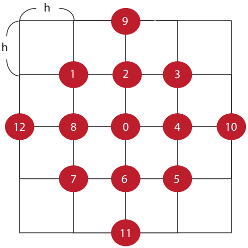

Figure: An example of interconnection structure

Modified equation of motion

In the previous figure, 2D Taylor expansions of each node with respect to the node 0 yeild

$$
\begin{aligned}
&\sum_{k=0}^{12} a_k w(x_0+\bar{x}_k, y_0+\bar{y}_k)\\
&= w(x_0,y_0)\sum_{k=0}^{12} a_k + \left(\frac{\partial w(x_0,y_0)}{\partial y}\sum_{k=0}^{12} a_k\bar{y}_k + \frac{\partial w(x_0,y_0)}{\partial x}\sum_{k=0}^{12} a_k\bar{x}_k\right)\\
&+\frac{1}{2}\left(\frac{\partial^2 w(x_0,y_0)}{\partial y^2}\sum_{k=0}^{12} a_k\bar{y}_k^2 + 2\frac{\partial^2 w(x_0,y_0)}{\partial x\partial y}\sum_{k=0}^{12} a_k\bar{x}_k\bar{y}_k + \frac{\partial^2 w(x_0,y_0)}{\partial x^2}\sum_{k=0}^{12} a_k\bar{x}_k^2\right)\\
&+\frac{1}{6}\left(\frac{\partial^3 w(x_0,y_0)}{\partial y^3}\sum_{k=0}^{12} a_k\bar{y}_k^3 + 3\frac{\partial^3 w(x_0,y_0)}{\partial x\partial y^2}\sum_{k=0}^{12} a_k\bar{x}_k\bar{y}_k^2 + 3\frac{\partial^3 w(x_0,y_0)}{\partial x^2\partial y}\sum_{k=0}^{12} a_k\bar{x}_k^2\bar{y}_k + \frac{\partial^3 w(x_0,y_0)}{\partial x^3}\sum_{k=0}^{12} a_k\bar{x}_k^3\right)\\
&+\frac{1}{24}\left(\frac{\partial^4 w(x_0,y_0)}{\partial y^4}\sum_{k=0}^{12} a_k\bar{y}_k^4 + 4\frac{\partial^4 w(x_0,y_0)}{\partial x\partial y^3}\sum_{k=0}^{12} a_k\bar{x}_k\bar{y}_k^3 + 6\frac{\partial^4 w(x_0,y_0)}{\partial x^2\partial y^2}\sum_{k=0}^{12} a_k\bar{x}_k^2\bar{y}_k^2\right.\\
&\quad\left. + 4\frac{\partial^4 w(x_0,y_0)}{\partial x^3\partial y}\sum_{k=0}^{12} a_k\bar{x}_k^3\bar{y}_k + \frac{\partial^4 w(x_0,y_0)}{\partial x^4}\sum_{k=0}^{12} a_k\bar{x}_k^4\right) + O(h^5).
\end{aligned}
\tag{4}
$$

Modified equation of motion

From the equation (4), we can see that the partial derivative term can be linearized as

$$\sum_{k=0}^{12} a_k w(x_0+\bar{x}_k, y_0+\bar{y}_k) \approx \left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial^2 x \partial^2 y}+\frac{\partial^4 w}{\partial^4 y}\right). \tag{5}$$

Then we can obtain the spatially discretized equation of motion as

$$\sigma d \frac{dw(t,\, x_i,\, y_j)^2}{d^2 t} + \underbrace{D\left(\sum_{k=0}^{12} a_k w(t, x_i+\bar{x}_k, y_j+\bar{y}_k)\right)}_{\text{interconection term}} = u(t,\, x_i,\, y_j). \tag{6}$$

Modified equation of motion

The parameters can be identified using the Vandermonde matrix.

$$\begin{bmatrix} 1 & \dots & 1 \\ \bar{x}_0 & \dots & \bar{x}_{12} \\ \bar{y}_0 & \dots & \bar{y}_{12} \\ \bar{x}_0^2 & \dots & \bar{x}_{12}^2 \\ \bar{y}_0^2 & \dots & \bar{y}_{12}^2 \\ \bar{x}_0\bar{y}_0 & \dots & \bar{x}_{12}\bar{y}_{12} \\ \bar{x}_0^3 & \dots & \bar{x}_{12}^3 \\ \bar{y}_0^3 & \dots & \bar{y}_{12}^3 \\ \bar{x}_0^2\bar{y}_0 & \dots & \bar{x}_{12}^2\bar{y}_{12} \\ \bar{x}_0\bar{y}_0^2 & \dots & \bar{x}_{12}\bar{y}_{12}^2 \\ \bar{x}_0^4 & \dots & \bar{x}_{12}^4 \\ \bar{y}_0^4 & \dots & \bar{y}_{12}^4 \\ \bar{x}_0^2\bar{y}_0^2 & \dots & \bar{x}_{12}^2\bar{y}_{12}^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 24 \\ 24 \\ 8 \end{bmatrix} \rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix} = \frac{1}{h^4} \begin{bmatrix} 20 \\ 2 \\ -8 \\ 2 \\ -8 \\ 2 \\ -8 \\ 2 \\ -8 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{7}$$

Modified equation of motion(State-space form)

By replacing the spatial domain (x, y) with (s_1, s_2) and introducint the spatial operator S_1 and S_2 which have the featuers

$$\begin{array}{c} S_1 w(t, s_1, s_2) = w(t, s_1 + 1, s_2), S_1^{-1} w(t, s_1, s_2) = w(t, s_1 - 1, s_2) \\ S_2 w(t, s_1, s_2) = w(t, s_1, s_2 + 1), S_2^{-1} w(t, s_1, s_2) = w(t, s_1, s_2 - 1), \end{array} \tag{8}$$

the equation described by (6) can be transformed as

$$\begin{array}{l} \sigma d \ddot{w}\left(t, s_1, s_2\right) + \dfrac{D}{h^4}\left(20 + 2\left(S_1^{-1} S_2 + S_1 S_2 + S_1 S_2^{-1} + S_1^{-1} S_2^{-1}\right)\right. \\ \left. -8\left(S_2 + S_1 + S_2^{-1} + S_1^{-1}\right) + \left(S_2 S_2 + S_1 S_1 + S_2^{-1} S_2^{-1} + S_1^{-1} S_1^{-1}\right)\right) w = u. \end{array} \tag{9}$$

Modified equation of motion(State-space form)

By defining the state variables

$$\mathrm{x} = \begin{bmatrix} \mathrm{x}_1(t, s_1, s_2) \\ \mathrm{x}_2(t, s_1, s_2) \end{bmatrix} = \begin{bmatrix} w(t, s_1, s_2) \\ \dot{w}(t, s_1, s_2) \end{bmatrix}, \tag{10}$$

the equation of motion can be represented as the spatially interconnected form[2] as

$$\dot{x} = A_{TT}x(t, s_1, s_2) + A_{TS}v(t, s_1, s_2) + B_T u(t, s_1, s_2) \tag{11}$$

$$v = A_{ST}x(t, s_1, s_2) + A_{SS}v(t, s_1, s_2) + B_S u(t, s_1, s_2) \tag{12}$$

$$y = C_T x(t, s_1, s_2) + C_S v(t, s_1, s_2). \tag{13}$$

[2] R. D'Andrea and G. E. Dullerud, "Distributed control of spatially interconnected systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 9, pp. 1478–1495, Sep.2003.

where

$$A_{TT} = \begin{bmatrix} 0 & 1 \\ 20\alpha & 0 \end{bmatrix} \text{ where } \alpha = -\frac{D}{\sigma d h^4} \tag{14}$$

$$A_{TS} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -8 & 2 & -8 & 2 & -8 & 2 & -8 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{15}$$

$$A_{ST} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{16}$$

$$A_{SS} = [0] \tag{17}$$

$$B_T = \begin{bmatrix} 0 \\ \frac{1}{\sigma d} \end{bmatrix} \tag{18}$$

$$B_S = [0] \tag{19}$$

$$C_T = 1 \tag{20}$$

$$C_S = 0. \tag{21}$$

Problem formulation

- Subunit model:

$$x_i^j(k+1) = A_{TT}^j x_i^j(k) + A_{TS}^j v_i^j(k) + B_{Tu}^j u_i^j(k) \tag{22}$$
$$w_i^j(k) = A_{ST}^j x_i^j(k) + A_{SS}^j v_i^j(k) + B_{Su}^j u_i^j(k) \tag{23}$$
$$y_i^j(k) = C_T^j x_i^j(k) + C_S^j v_i^j(k) + D_u^j u_i^j(k) \tag{24}$$
$$x_i^j(0) = x_0^j, \text{ for } k = 0, 1, \dots, K. \tag{25}$$

  where $j \in \mathscr{V}$, i is the iteration number, $x_i^j(k) \in \mathbb{R}^{m_j}$ is the state, $v_i^j(k) \in \mathbb{R}^{n_j}$ is the interconnected input, $u_i^j(k) \in \mathbb{R}^{r_j}$ is the control input, $w_i^j(k) \in \mathbb{R}^{n_j}$ is the interconnected output, and $y_i^j(k) \in \mathbb{R}^{r_j}$ is the output at the subunit j.
- Interconnection topology: $\mathscr{G} = (\mathscr{V}, \mathscr{E})$.

Modified state-space model

The variables in (22) – (25) can be rewritten as the super-vector containing information at the whole time instants

$$\mathbf{X}_i^j = \begin{bmatrix} x_i^j(1) & x_i^j(2) & \cdots & x_i^j(K) \end{bmatrix}^T \tag{26}$$

$$\mathscr{V}_i^j = \begin{bmatrix} v_i^j(0) & v_i^j(1) & \cdots & v_i^j(K) \end{bmatrix}^T \tag{27}$$

$$\mathbf{W}_i^j = \begin{bmatrix} w_i^j(0) & w_i^j(1) & \cdots & w_i^j(K) \end{bmatrix}^T \tag{28}$$

$$\mathbf{U}_i^j = \begin{bmatrix} u_i^j(0) & u_i^j(1) & \cdots & u_i^j(K-1) \end{bmatrix}^T \tag{29}$$

$$\mathbf{Y}_i^j = \begin{bmatrix} y_i^j(0) & y_i^j(1) & \cdots & y_i^j(K-1) \end{bmatrix}^T \tag{30}$$

Modified state-space model

In addition, introduce so-called stacking vectors containing information at both temporal and sptial position

$$\tilde{\mathbf{X}}_i = \begin{bmatrix} \mathbf{X}_i^1 & \mathbf{X}_i^2 & \cdots & \mathbf{X}_i^L \end{bmatrix}^T \tag{31}$$

$$\tilde{\mathbf{V}}_i = \begin{bmatrix} \mathscr{V}_i^1 & \mathscr{V}_i^2 & \cdots & \mathscr{V}_i^L \end{bmatrix}^T \tag{32}$$

$$\tilde{\mathbf{W}}_i = \begin{bmatrix} \mathbf{W}_i^1 & \mathbf{W}_i^2 & \cdots & \mathbf{W}_i^L \end{bmatrix}^T \tag{33}$$

$$\tilde{\mathbf{U}}_i = \begin{bmatrix} \mathbf{U}_i^1 & \mathbf{U}_i^2 & \cdots & \mathbf{U}_i^L \end{bmatrix}^T \tag{34}$$

$$\tilde{\mathbf{Y}}_i = \begin{bmatrix} \mathbf{Y}_i^1 & \mathbf{Y}_i^2 & \cdots & \mathbf{Y}_i^L \end{bmatrix}^T \tag{35}$$

Modified state-space model

Then the overall relation among the subsystems can be expressed by

$$\tilde{\mathbf{X}}_i = \tilde{\mathbf{A}}_{TT}\tilde{\mathbf{X}}_0 + \tilde{\mathbf{A}}_{TS}\tilde{\mathbf{V}}_i + \tilde{\mathbf{B}}_{Tu}\tilde{\mathbf{U}}_i \tag{36}$$

$$\tilde{\mathbf{W}}_i = \tilde{\mathbf{A}}_{ST}\tilde{\mathbf{X}}_0 + \tilde{\mathbf{A}}_{SS}\tilde{\mathbf{V}}_i + \tilde{\mathbf{B}}_{Su}\tilde{\mathbf{U}}_i \tag{37}$$

$$\tilde{\mathbf{Y}}_i = \tilde{\mathbf{C}}_{T}\tilde{\mathbf{X}}_0 + \tilde{\mathbf{C}}_{S}\tilde{\mathbf{V}}_i + \tilde{\mathbf{D}}_{u}\tilde{\mathbf{U}}_i \tag{38}$$

Modified state-space model

where

$$\tilde{\mathbf{A}}_{TT} = diag[\hat{\mathbf{A}}^l_{TT}] \tag{39}$$

$$\tilde{\mathbf{A}}_{TS} = diag[\hat{\mathbf{A}}^l_{TS}] \tag{40}$$

$$\tilde{\mathbf{B}}_{Tu} = diag[\hat{\mathbf{B}}^l_{Tu}] \tag{41}$$

$$\tilde{\mathbf{A}}_{ST} = diag[\hat{\mathbf{A}}^l_{TS}] \tag{42}$$

$$\tilde{\mathbf{A}}_{SS} = diag[\hat{\mathbf{A}}^l_{SS}] \tag{43}$$

$$\tilde{\mathbf{B}}_{Su} = diag[\hat{\mathbf{B}}^l_{Su}] \tag{44}$$

$$\tilde{\mathbf{C}}_{T} = diag[\hat{\mathbf{C}}^l_{T}] \tag{45}$$

$$\tilde{\mathbf{C}}_{S} = diag[\hat{\mathbf{C}}^l_{S}] \tag{46}$$

$$\tilde{\mathbf{D}}_{u} = diag[\hat{\mathbf{D}}^l_{TT}] \tag{47}$$

for $l = 1, 2, \ldots, L$.

Control strategy

Theorem

Consider the equations represented by (36), (37), *and* (38), *which describe the whole system comprising subsystems with identical initial conditions. For a finite time interval* $k = 0, 1, 2, ..., K$, *the desired trajectory is given by* $y_d^j(k)$ *which can be transformed into* $\tilde{\mathbf{Y}}_d$ *in a stacking vector form with the decentralized ILC law described by*

$$\tilde{\mathbf{U}}_{i+1} = \tilde{\mathbf{U}}_i + \mathbf{T}_e(\tilde{\mathbf{Y}}_d - \tilde{\mathbf{Y}}_i) \tag{48}$$

where

$$\mathbf{T}_e = \begin{bmatrix} z_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & z_q \end{bmatrix}. \tag{49}$$

Theorem

(continue)
For such a system, the error $\mathscr{E}^* = \tilde{\mathbf{Y}}_d - \tilde{\mathbf{Y}}_i$ *converges to zero asymptotically if there exists a diagonal matrix* $\mathbf{T}_e$ *such that the inequality is satisfied as*

$$\rho(\mathbf{I} - \mathbf{T}_e \mathbf{T}_S) < 1. \tag{50}$$

where $\rho(\cdot)$ *denotes the spectral radius.*

Simulation

Suppose that there are three subunits($j = 1,2,3$) for a finite time instants described by $k = [1,2,...,50](sec)$. The state-space models of the three subunits are numerically given as

$$\left[\begin{array}{c|c|c} A^j_{TT} & A^j_{TS} & B^j_{Tu} \\ \hline A^j_{ST} & A^j_{SS} & 0 \\ \hline C^j_T & 0 & 0 \end{array}\right] \tag{51}$$

$$x_i(0) = 0 \tag{52}$$

where

$$A^j_{TT} = -j \tag{53}$$

$$A^j_{TS} = \left[\begin{array}{ccc} j & j & j \end{array}\right] \tag{54}$$

$$B^j_{Tu} = 1 \tag{55}$$

$$A^j_{ST} = \left[\begin{array}{ccc} j & j & j \end{array}\right]^T \tag{56}$$

$$A^j_{SS} = diag\{0.1 \times j\} \tag{57}$$

$$C^j_T = 0.5 \text{ for j=1,2,3.} \tag{58}$$

Simulation

The desired trajectory vector is given for the time instant $k = 1, 2, 3, ..., 50(sec)$,

$$\begin{bmatrix} y_d^1(k) \\ y_d^2(k) \\ y_d^3(k) \end{bmatrix} = \begin{bmatrix} \cos(0.1k) \\ k(k+1) \\ \tan(k)^2 \end{bmatrix} \tag{59}$$

Simulation

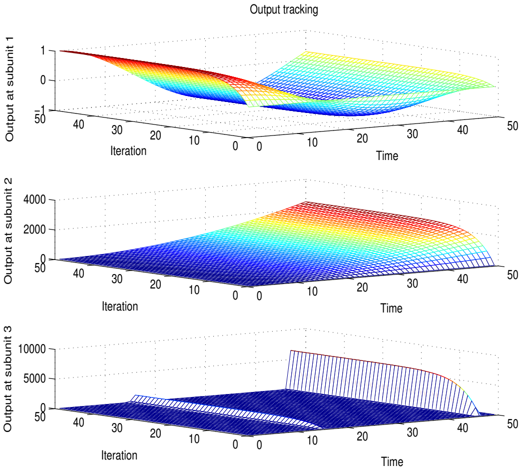

Figure: Output tracking performance

Simulation

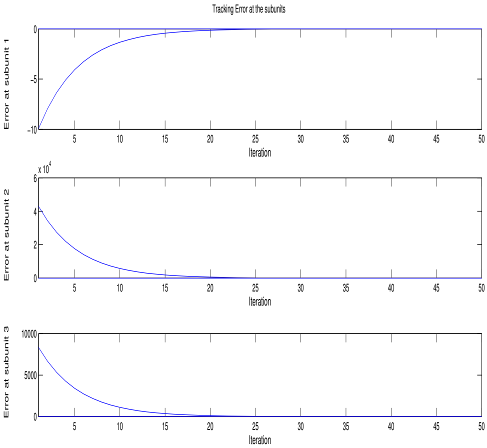

Figure: Error convergence

Conclusion

- We proposed a decentralized iterative learning controller for a spatially heterogeneous system with arbitrary interconnections.
- As shown from the numerical simulation results, the tracking error converges to zero as the iterative number increases.
- As a future work, it is desirable to get more realistic model to decribe the whole adpative optics system.

Thank you for your attention.

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