Reliable Distributed Estimation with Intermittent Communications

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Introduction

- Authors were motivated by developing distributed algorithms for tracking multiple objects using passive sensors.

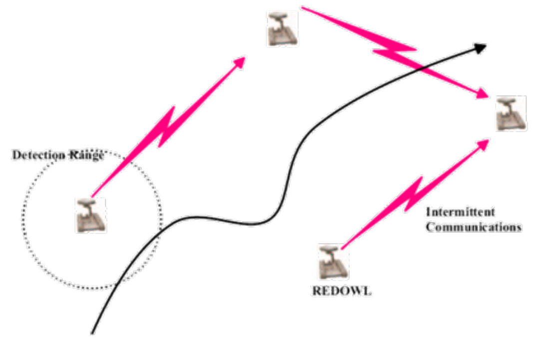


Fig. 1. Target Tracking with REDOWLs

- The main difficulty arises from **optimally fusing the intermittent local statistics** received at the fusion center.

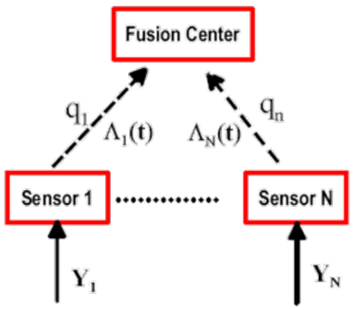


Fig. 2. Schematic illustration of the setup a sensor network. Any sensor or all sensors can serve as fusion centers. Sensors are connected through lossy communications links.

Problem Statement (cont'd)

- Discrete-time system

$$X_{t+1} = AX_t + W_t, \quad X_0 \sim N(0, \Sigma_0), \qquad (1)$$

- Measurement of sensor $v \in V$

$$Y_t(v) = C_t(v)X_t + U_t(v), \qquad v \in V,$$

- When all measurements are immediately available to a central processor, MMSE estimator is

$$X_{t|t} = E[X_t | Y_\tau(v) : v \in V, \tau \leq t] \text{ of } X_t \text{ is}$$

$$X_{t|t} = X_{t|t-1} + P_{t|t} \sum_{v \in V} C_t^T(v) \Sigma_U^{-1} (Y_t(v) - C(v) X_{t|t-1}) \quad (2)$$

Problem Statement

- Definition 2.1
  - A decentralized fusion strategy is said to be **anytime optimal** <u>if the fusion center at any time, t, can realize the following estimate</u>:

$$X_{t|\mathbf{N}(t)} = E(X(t) \mid \mathcal{Y}_v^{N_v(t)}, \forall v \in V)$$

  - N(t) is the vector of arrival times. $\mathbf{N}(t) = (N_1(t), N_2(t), \ldots N_{|V|}(t))$
  - $\mathcal{Y}_v^k$ denote sensor v's measurement up to time k

Local Processing Strategies

- In the special case W(t) = 0 in (1), the resulting problem becomes a **conditionally deterministic system**.

- The local estimates are

$$(P^v_{t|t})^{-1} X^v(t|t) = (P^v_{t|t-1})^{-1} X^v(t|t-1) + C^T_t(v)\Sigma_U^{-1} Y_t(v)$$
$$(P^v_{t|t})^{-1} = (P^v_{t|t-1})^{-1} + C^T_t(v)\Sigma_U^{-1} C_t(v) \qquad (4)$$

- The local predict rule is

$$X^v(t+1|t) = AX^v(t|t);\; P^v_{t+1|t} = AP^v_{t|t}A^T + \Sigma_W \qquad (5)$$

Fusion Algorithm

- $X_{t|\mathbf{N}(t)}$ and $P_{t|\mathbf{N}(t)}$ are the **centralized** estimate and error covariance.
- $X^v(t|N_v(t)), P^v_{t|N_v(t)}$ are the **local predicted** estimate and error covariance.
- The first result specifies the optimal algorithm at the fusion center:

Theorem III.1 *Assume that the fusion center has received communications $X^v(N_v(t)|N_v(t))$ from sensors $v \in V$ at times $N_v(t) \leq t$. Then the following decentralized fusion rule achieves anytime optimality, i.e.,*

$$X(t|\mathbf{N}(t)) = P_{t|\mathbf{N}(t)} \sum_{v \in V} (P^v_{t|N_v(t)})^{-1} X^v(t|N_v(t)) \tag{6}$$

$$P^{-1}_{t|\mathbf{N}(t)} = \sum_{v \in V} (P^v_{t|N_v(t)})^{-1} - (|V|-1)(P^0_{t|t})^{-1} \tag{7}$$

Two Sensor Case with Two way Communication

- The process noise is no longer zero with two sensors.
- The global KF estimate can be written:
  - A single remote sensor is denoted as l, and a fusion center sensor is denoted as f.

$$P_{t|t}^{-1}X_{t|t} = P_{t|t-1}^{-1}X_{t|t-1} + \sum_{v\in\{l,f\}} C_t^\top(v)(R^v)^{-1}Y_t^v$$

$$= \boxed{P_{t|t-1}^{-1}AP_{t-1|t-1}}\boxed{P_{t-1|t-1}^{-1}X_{t-1|t-1}} + \sum_{v\in\{l,f\}} C_t^\top(v)(R^v)^{-1}Y_t^v$$

Let $S(t) = \boxed{P_{t|t}^{-1}X_{t|t}}$ and $\tilde{A}(t) = \boxed{P_{t|t-1}^{-1}AP_{t-1|t-1}}$ to get:

$$S(t) = \tilde{A}(t)S_{t-1} + \sum_{v\in\{l,f\}} C_t^\top(v)(R^v)^{-1}Y_t^v \qquad (8)$$

  - S(t) is the state evolution of the information.

Fusion Algorithms Considering Packet Loss

- When no messages are received from the local sensor, the fusion center can use its measurements to **update the state estimate X(t|N(t))**.
- Whenever the local sensor succeeds, it transmits S_t^l to the fusion center.
- $\sigma_1, \ldots, \sigma_n$ is a sequence of time instants of successful transmission, and then $t \in (\sigma_j, \sigma_{j+1})$.

$$
\begin{aligned}
S^l(t) &= \tilde{A}(t)S^l(t-1) + C_t^\top(l)(R^l)^{-1}Y_t^l, \ S^l(\lfloor\sigma_j\rfloor) = 0 \\
S^f(t) &= \tilde{A}(t)S^f(t-1) + C_t^\top(f)(R^f)^{-1}Y_t^f \\
S^f(\lfloor\sigma_j\rfloor) &= P^{-1}_{\lfloor\sigma_j\rfloor|\lfloor\sigma_j\rfloor} X_{\lfloor\sigma_j\rfloor|\lfloor\sigma_j\rfloor} \qquad (9)
\end{aligned}
$$

- The decentralized fusion strategy outlined in (9) achieves **anytime optimality.**

Simulation Result

- Figure shows the results of an experiments with two sensors communicating to a fusion center.

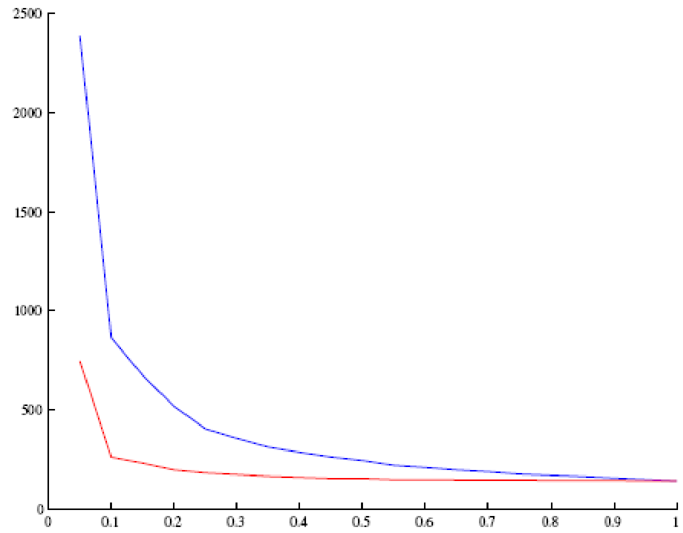
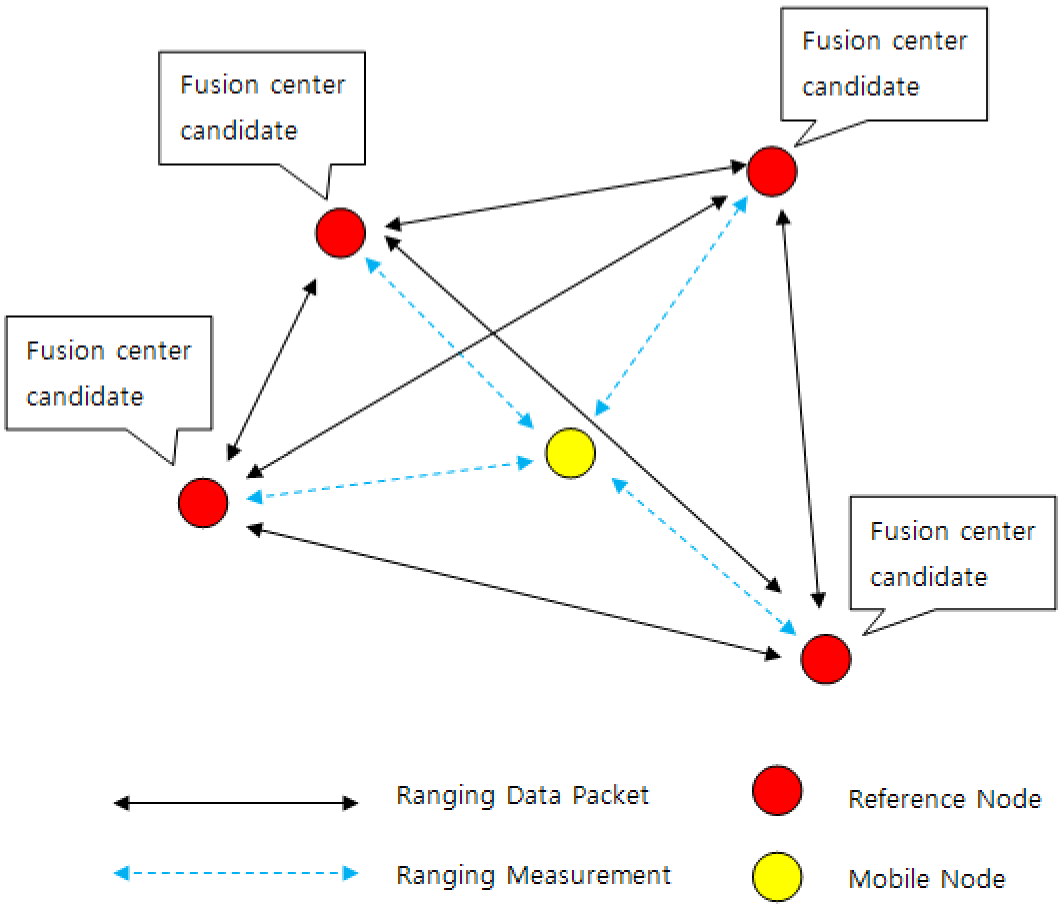


Fig. 4. Trace of steady state error covariance versus communication success probability. Upper curve is measurement transmission, lower curve is estimate transmission.

Relation with My Research



Thank!!!

- Q&A